MATHEMATICAL MODEL OF A PLASTIC TUBULAR HEAT EXCHANGER

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The article presents a newly devised mathematical model describing the process of heat exchange in a plastic tubular exchanger. On the basis of this model a program for calculating the indices of the heat exchanger was run. The dependence of the thermal efficiency of the plastic heat exchanger on the outer diameter of the tubes was analyzed.

Reducing the metal content of products is a very important tsk of engineering. The use of polymer materials is one of the ways of solving this task. A promising trend in this field is the construction of plastic heat exchangers. Heat exchangers made of plastics have the following advantages: low weight, cheapness, technological suitability, slight pollution, easy cleaning, corrosion resistance, resistance to aggressive media [1-4]. For plastic heat exchangers it is expedient to use a design with coefficient of ribbing close to unity and developed cooling surfaces on which heat exchange between the heat carriers proceeds solely through a comparatively thin wall [5]. These requirements are due to the low thermal conductivity of plastics. The tubular exchanger fulfills the mentioned conditions, and also the condition of technological suitability in production.

Let us consider the tubular exchanger suggested in [6]. The matrix of the heat exchanger consists of circular plastic tubes joined together by plastic plates (Fig. 1). The cooled liquid (e.g., water) moves through the tubes, the cooling air moves in the space between the tubes in a direction perpendicular to the movement of the liquid. Figure 2 presents a cutout of the section of the heat exchanger by a plane parallel to the plates.

On the basis of [7] we worked out a mathematical model describing the process of heat exchange in the heat exchanger under consideration. The mathematical model is based on dividing the matrix of the plastic heat exchanger into calculation cells each of which consists of a cutout of a tube and of the space surrounding it. The cell, in turn, is divided into sections containing the front and rear parts of the tube. We determine the heat removal from the initial section, its aerodynamic and hydraulic resistance. Then we go over to the examination of the subsequent section, the subsequent cell, etc. The parameters of the heat carriers at the inlet to the calculation cell (section) are adopted equal to

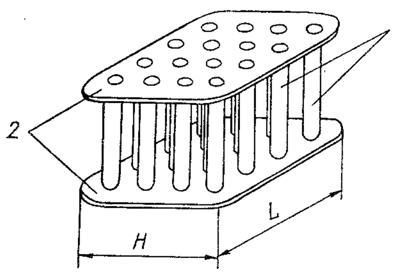

Fig. 1

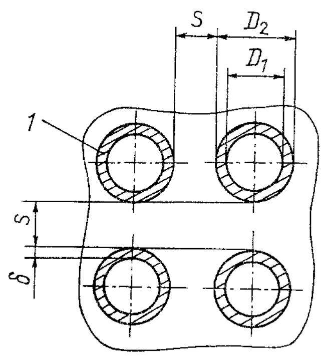

Fig. 2

Fig. 1. Cooling matrix of the heat exchanger: 1) tubes; 2) plates.

Fig. 2. Cutout of a section of the matrix: 1) tubes.

Orenburg Polytechnic Institute. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 60, No. 2, pp. 297-302, February, 1991. Original article submitted April 17, 1990.

the parameters of the heat carriers at the outlet from the preceding cells (sections). After the calculation has been carried out over the entire volume of the heat exchanger, we know its thermal efficiency, the aerodynamic and hydraulic resistances. Heat removal from the calculation section is determined in dependence on the magnitude of heat transfer from the side of the heat carriers, their temperature, thermal conductivity, and wall thickness of the tube. The aerodynamic resistance of the heat exchanger is the sum of the pressure losses on the narrowing and widening of the air stream in the flow around the tubes, the losses on friction and heating. The hydraulic losses of the liquid moving through the tube are composed of the pressure losses due to the sudden narrowing of the flow where it enters the tube, flow losses along the tube, and losses on the sudden widening of the flow at its exit from the tube.

On the basis of the mathematical model we worked out a computer program for calculating the indices of the plastic tubular exchanger. The initial data for the calculation are: the overall dimensions of the heat exchanger, the distances between tubes within a row and between rows of tubes, the outer and inner diameters of the tube, the air flow rate through the heat exchanger or the speed of the air at its inlet, the pressure and temperature of the incoming air stream, the water flow rate through the heat exchanger, the water temperature at its inlet, the density and thermal conductivity of the material of the tubes. The result of the operation of the program are the values of the weight, the thermal efficiency, the aerodynamic and hydraulic resistances of the calculated heat exchanger. The calculations carried out with the aid of this program made it possible to determine the nature of the dependences of the thermal efficiency, of the aerodynamic and hydraulic resistances of the plastic heat exchanger on its geometric dimensions, the parameters of the heat carriers, the thermal conductivity of the material of the tubes.

The most important geometric dimensions of the heat exchanger affecting the process of heat exchange in it are the outer diameter of the tubes and the distance between them. By varying these values we can obtain different values of thermal efficiency of the heat exchanger with unchanged overall dimensions and unchanged parameters of the heat carriers at the inlet to the heat exchanger. To attain the specified thermal efficiency by running through different variants requires considerable computing time. To make the calculations less laborious it is necessary to choose such a ratio between the outer diameter of the tubes and the distance between them at which the thermal efficiency of the heat exchanger is close to the maximally possible one. For that purpose we examine the dependence of the thermal efficiency of heat exchangers on the outer diameter of the tubes. The overall dimensions of the heat exchanger, the parameters of the heat carriers at its inlet, the wall thickness of the tubes, the distance between tubes are regarded as specified. It is taken into account that a change of the outer tube diameter leads to a change of: a) the thermal resistance of the tube; b) the heat transfer from the side of the air and of the water; c) the number of tubes in the heat exchanger.

The following assumptions were made:

1) heat removal from each pipe of the heat exchanger is the same;

2) the wall thickness of the tube is fairly small in comparison with its diameter.

It is obvious that the maximal thermal efficiency of the heat exchanger is attained when the value of R is minimal; this is the reciprocal value of the heat transfer coefficient between the two heat carriers. With a view to the adopted assumptions the value of R is determined in the following way:

$$R = \frac{(S + D_2)^2}{HL}\left(\frac{1}{\alpha_1 D_1} + \frac{\delta}{\lambda D_1} + \frac{1}{\alpha_2 D_2}\right). \quad (1)$$

Taking into account assumption 2), and also the fact that the first term of formula (1) is one order of magnitude smaller than the others because of the larger heat transfer coefficient from the side of the liquid, we adopt

$$D_1 = D_2 = D. \quad (2)$$

The dependence of the heat transfer coefficient from the side of the air on the outer diameter of the tubes has the following form:

$$\alpha_2 = C\frac{1}{D}(DS + D^2)^{2/3}, \tag{3}$$

where

$$C = 0{,}26\,\lambda_a \mathrm{Pr}_a^{1/3}\left(\frac{\rho_a}{\mu_a}\right)^{2/3}\left(\frac{W_a}{S}\right)^{2/3}.$$

In deriving expression (3) we determined the Nusselt number by the formula for heat transfer in transverse washing of the tube bundles in the tube spaces [7]

$$\mathrm{Nu}_в = 0{,}26\,\mathrm{Re}_a^{2/3}\mathrm{Pr}_a^{1/3},$$

where for convenience of the transformations the exponent of the Reynolds number was taken equal to 2/3, not to 0.65, which does not introduce a large error. The characteristic dimension is the outer tube diameter. In determining the Reynolds number we used the speed of the air at the narrowest place in the space between the tubes. With (2) and (3) taken into account, formula (1) assumes the form

$$R = \frac{1}{HL}\left[\frac{1}{\alpha_1}\frac{(S+D)^2}{D} + \frac{\delta}{\lambda}\frac{(S+D)^2}{D} + \right.$$
$$\left. + \frac{1}{C}\left(\frac{(S+D)^2}{D}\right)^{2/3}\right].$$

We find the outer tube diameter at which the value of R is minimal. At first we assume that the heat transfer coefficient on the side of the water does not depend on the diameter of the tubes. The derivative of R with respect to the tube diameter is determined:

$$\frac{dR}{dD} = \frac{1}{HL}\left(1 - \frac{S^2}{D^2}\right)\left(\frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{2}{3C}\frac{D^{1/3}}{(S+D)^{2/3}}\right).$$

On condition that D = S is the first derivative of R, is equal to zero, and the second derivative is positive:

$$\frac{d^2R}{dD^2} = \frac{2}{HLS}\left[\frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{2}{3C}(4S)^{-1/3}\right] > 0.$$

Consequently, the function R(D) has a minimum at the point D = S. The nature of the dependence of R on D is illustrated in Fig. 3 (curve 1).

As a rule, heat transfer from the side of the liquid far exceeds the heat transfer from the side of the air. The change of the heat transfer coefficient from the liquid to the tubes in consequence of the changed tube diameter has no substantial effect on the thermal efficiency of the heat exchanger. We therefore evaluate only what effect the dependence of the heat transfer coefficient from the side of the water on the tube diameter has on the value of R. For that we consider the transformed expression (1)

$$R = \frac{1}{HL}\frac{(S+D)^2}{\alpha_1 D}. \tag{4}$$

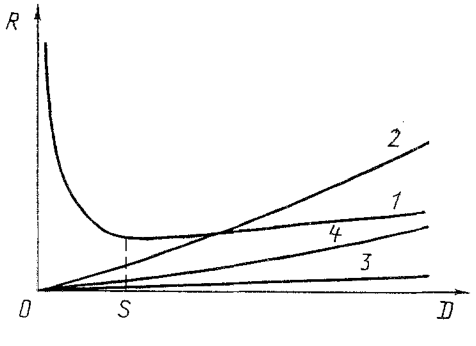

Fig. 3. Nature of the dependence R(D): 1) α_1 = const; 2) n = 0.4; 3) 1.24; 4) 0.8.

The heat transfer of water is calculated by the formulas for the case of forced liquid flow in tubes [7]

$$\mathrm{Nu}_{\ell} = A\,\mathrm{Re}_{\ell}^{n}\mathrm{Pr}_{\ell}^{m}(\mathrm{Pr}_{\ell}/\mathrm{Pr}_{\ell w})^{k}. \tag{5}$$

In formula (5) we select the constants A, n, m in dependence on the flow conditions of the liquid in the tube, and we adopt the ratio $\mathrm{Pr}_{\ell}/\mathrm{Pr}_{\ell w}$ equal to unity. The characteristic dimension is the inner diameter of the tube. When determining the Reynolds number we use the speed of the liquid in the tube. Then the heat transfer coefficient on the side of the liquid is determined as

$$\alpha_1 = B\frac{1}{D}\left[\frac{(S+D)^2}{D}\right]^n, \tag{6}$$

where

$$B = A\lambda_{\ell}\mathrm{Pr}_{\ell}^{m}\left(\frac{\rho_{\ell}}{\mu_{\ell}}\right)^n\left(\frac{4G}{\pi LH}\right)^n.$$

With a view to (6) expression (4) assumes the form

$$R = \frac{1}{HLB}D^n(S+D)^{2-2n}.$$

In this case the nature of the change of the value of R is described by curves 2-4 in Fig. 3. Curve 2 describes the change of R in laminar flow of liquid in tubes, the exponent of the Reynolds number is n = 0.4. Curve 3 corresponds to the transient regime of liquid flow, the exponent of the Reynolds number is n = 1.24, and curve 4 corresponds to turbulent liquid flow, n = 0.8.

It can be seen from Fig. 3 that an increase of the outer tube diameter to more than the value of S leads to increased R in any liquid flow regime. A reduction of the diameter of the tubes to less than S leads to some decrease of R, and then to its increase to an infinitely large value.

Thus that heat exchanger has maximal thermal efficiency in which the outer diameter of the tubes is equal to or somewhat smaller than the distance between the tubes. The results of calculations according to the newly devised computer program confirm this conclusion. Figure 4 shows the dependences of the thermal efficiency of a plastic heat exchanger on the outer diameter of the tubes with different distances between them. The heat exchanger has the overall dimensions 0.749 × 0.706 × 0.093 m, the thermal conductivity of the plastic

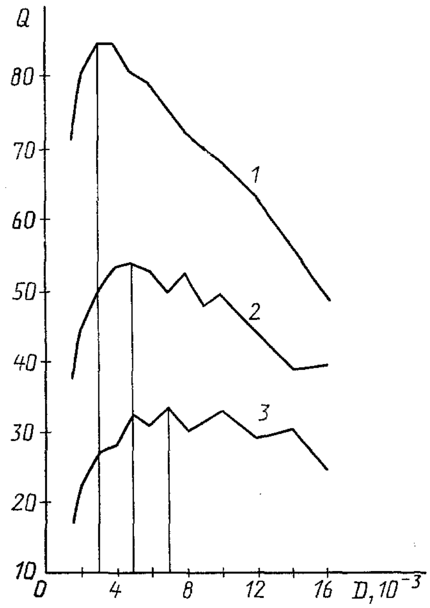

Fig. 4. Dependence of the thermal efficiency of the plastic heat exchanger on the outer diameter of the tubes: 1) S = 0.003 m; 2) 0.005; 3) 0.008. D, m; Q, kW.

was taken to be equal to 0.2 W/(m·K), the wall thickness of the tube is 0.005 m. The speed of the air at the inlet to the heat exchanger is 6 m/sec, the water flow rate is 3.225 kg/sec. It can be seen from the figure that thermal efficiency is maximal with tube diameters of 3, 5, and 7 mm, with distances between the tubes equal to 3, 5, and 8 mm, respectively, i.e., when the outer diameter of the tubes is equal to the distance between them. The dependence of the thermal efficiency of the heat exchanger on the tube diameter is neither smooth nor ambiguous. The general tendency toward improved thermal efficiency with tube diameter increased to the value of the distance between the tubes, and the tendency to impaired thermal efficiency with further increase of the tube diameter are retained. However, at the same time there occurs local impairment and improvement of thermal efficiency when the diameter is changed because a change of the number of tubes in the heat exchanger occurs jumpwise, and consequently the area of the heat exchanger surface also changes jumpwise. This, in particular, explains why the maximum of thermal efficiency may correspond to a tube diameter that somewhat differs from the distance between the tubes.

Notation. R) reciprocal value of the heat transfer coefficient between two heat carriers; s) distance between tubes; D_2, D) outer tube diameter; D_1) inner tube diameter; δ) wall thickness of the tube; λ) thermal conductivity of the material of the tube; H, L) width and length of the heat exchanger matrix, respectively; α_1, α_2) heat transfer coefficient from the side of the liquid and air, respectively; ρ_a, μ_a, λ_a) density, dynamic viscosity, and thermal conductivity of air, respectively; W_a) speed of air at the inlet to the heat exchanger; Pr_a, Nu_a, Re_a) Prandtl, Nusselt, and Reynolds numbers, respectively, for air; Nu_ℓ, Re_ℓ) Nusselt and Reynolds numbers, respectively, for the liquid; Pr_ℓ, $Pr_{\ell w}$) Prandtl numbers for the liquid at the temperature of the liquid and of the tube wall, respectively; ρ_ℓ, μ_ℓ, λ_ℓ) density, dynamic viscosity, and thermal conductivity of the liquid, respectively, G) volume flow rate of liquid through the heat exchanger; Q) thermal efficiency of the heat exchanger. Subscripts: a) air; ℓ) liquid.

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